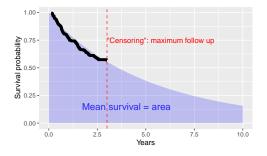
survextrap: a new tool for flexible and transparent extrapolation of survival data to inform health policy

Christopher Jackson

## Royal Statistical Society, Belfast, 6 December 2023



# Survival extrapolation: long-term decisions from short-term, time-to-event data



#### Examples:

Should health service adopt a new treatment, given 3 years follow up data from a trial?

Predicted burden on hospitals in an epidemic, given data on hospital stays, where many are people still in hospital?

Consequences of policy decisions will last longer than the end of the data

Christopher Jackson

## Policy-maker typically wants to know the expected time to event

- equivalent to knowing the total outcome (e.g. survival, hospital length of stay) over the population
- Not provided by most common survival analysis tools e.g. Kaplan-Meier estimators, Cox models.

Provided by a fully-parametric distribution for the time T to the event.

## Estimating expected survival over the long term

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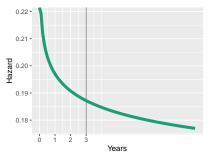
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Weibull distribution

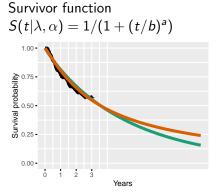
Survivor function

 $S(t|\lambda,\alpha) = \exp(-\lambda t^{\alpha})$ 

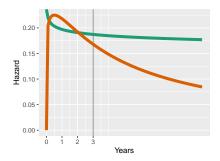
Hazard function  $h(t|\lambda, \alpha) = \lambda \alpha t^{\alpha-1}$ 

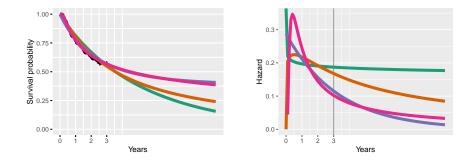


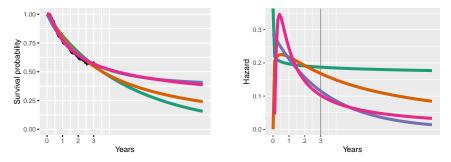
Log-logistic distribution



Hazard function  $h(t|\lambda, \alpha) =$ 







Fit a set of models and judge which :

- ▶ fits the data best in the short term? (easy, e.g. AIC)
- gives more plausible risk changes in the long term? harder

### Data on general population, or disease registry

survival of people with a specific disease cannot be better than a comparable set of people in the general population?

## Clinical judgements about the mechanism

 e.g. some people get cured of the disease, so disease-specific hazard of death reduces to zero

Formally-elicited judgements about, e.g. 5-year, 10-year survival?

Want to be able to use this kind of information in a transparent and statistically-principled way

# Reviews for methods for extrapolation with external data

#### Bullement et al. (Medical Decision Making 2023)

A Systematic Review of Methods to Incorporate External Evidence into Trial-Based Survival Extrapolations for Health Technology Assessment Medical Decision Making 1–11 © The Author(s) 2023 © O Article reuse guidelines: sugrpub.com/journals-permissions DOI: 10.1177/027298982331168618 journals.sugrbucem/home/mdm © SAGE

Ash Bullement<sup>(0)</sup>, Matthew D. Stevenson, Gianluca Baio, Gemma E. Shields<sup>(0)</sup>, and Nicholas R. Latimer

#### Jackson et al. (Medical Decision Making 2017)

## Extrapolating Survival from Randomized Trials Using External Data: A Review of Methods

```
Christopher Jackson, Pho, John Stevens, Pho, Shiljie Ren, Michil, Pho, Nick Latimer, Pho, Msc, Laura Show less A
Bojke, Pho, Msc, Andrea Manca, Pho, Msc, Linda Sharples, Pho
First Published July 10, 2016 | Research Article | Find in PubMed | @ Great transmission
https://doi.org.ze.lbi.cam.ac.uk/10.1177/0272883X16639900
```

#### NICE DSU (2021) https:

//www.sheffield.ac.uk/nice-dsu/tsds/flexible-methods-survival

 $\dots$  piecewise models, spline models, cure, relative survival, proportional and additive hazards, converging hazards, diminishing treatment effects, Bayesian methods  $\dots$ 

# Ideal characteristics of a method / tool

- 1. Incorporate all available data
- 2. Fit the data as well as possible
- 3. Make any assumptions transparent
- 4. Quantify uncertainty
- 5. Be easy to use!

In particular, Bayesian evidence synthesis methods

- are comprehensive, flexible, principled
- but have needed specialised programming (BUGS / JAGS), advanced statistical expertise

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## Package for Bayesian survival modelling

- Multiple sources of external data to aid extrapolation
- Flexible parametric spline model for the hazard
- Multiparameter evidence synthesis, MCMC estimation (Stan)
- Principle: data and influential assumptions made as transparent as possible.
  - "you say what you know, then the computer does the hard work of converting that to answers"!

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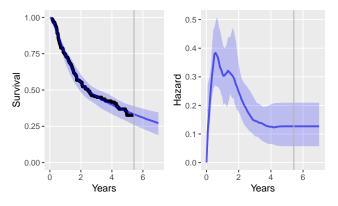
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# Basic survextrap model for one short-term dataset



Running example: head and neck cancer trial data from Guyot et al. 2017

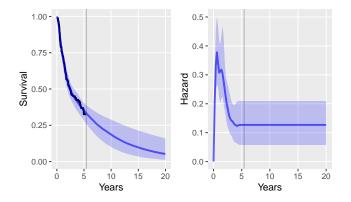
## Penalized M-spline model

Flexible hazard within data, constant beyond data by default

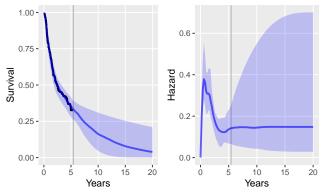
#### mod1 <- survextrap(Surv(years, d) ~ 1, data=dat)</pre>

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## Basic survextrap model for one short-term dataset



## Basic survextrap model for one short-term dataset



Allow hazard to vary up to 20 years, by adding a spline "knot". Assumes only that the hazard is smooth

short-term extrapolation influenced weakly by latest data.
Posterior distribution represents uncertainty due to lack of data
mod2 <- survextrap(Surv(years, d) ~ 1, add\_knots=20)</p>

Hazard h(t) is weighted average of basis functions, local to time periods defined by "knots"

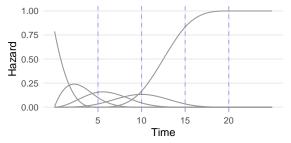
$$h(t) = \eta \sum_{i=1}^{n} p_i b_i(t)$$
  $\eta > 0, \sum p_i = 1$ 

Example: n = 5 basis functions  $b_i(t)$  shown as grey lines Parameters: weights  $p_i$  determining the shape.  $\eta$  is *scale* Hazard constant after the last knot

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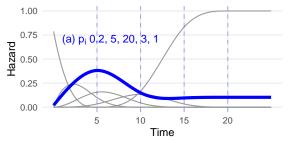
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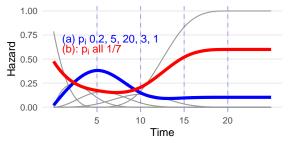
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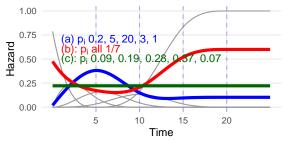
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- By default, knots chosen as quantiles of observed survival times, so similar amounts of information in each time interval.
- Recommendation: use lots of knots / basis coefficients
- Over-fitting controlled by a hierarchical model on basis coefficients p<sub>i</sub> (similar to "penalised likelihood").

 $p_i \sim MNlogit$  [mean: constant hazard; variance:  $\sigma$ ]

 $\sigma$  defines the smoothness of the hazard function  $\sigma = 0$ : constant hazard.  $\sigma$  higher  $\rightarrow$  more wiggly hazard  $\sigma$  given a prior, updated to posterior given data

#### Principle

learn how much smoothness is needed to represent the data
 "shrink" towards a constant hazard if data are weak

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Supplied as data frame of aggregate counts of survivors over a series of intervals, with covariates. Example:

Follow-up period		Number		Covariates	
From t	Тои	Alive at <i>t</i>	Still alive	Treatment	
			at u		
5	6	358	325	Control	
6	7	308	285	Control	
8	9	221	198	Control	
etc.		'			

# Use of external data (needed for the long term)

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- Registry data, population data, and elicited judgements can often be expressed in this form
- Each count of survivors is a binomal outcome, with probability defined by the spline model and covariates
- Bayesian evidence synthesis: posterior for parameters determined given individual and external data together

# Use of external data (needed for the long term)

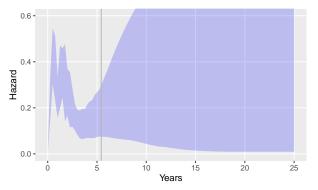
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Biased external data? — population with different risks from trial?

- Can model via a "dataset" covariate with proportional hazards (and/or other covariates) — assuming hazard shape common
  - can estimate hazard ratio if datasets observed at same times
  - but take care with extrapolating outside those times

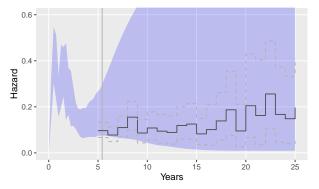
# Survival modelling with external data: example



With no external data, and just a smoothness assumption, estimated long-term hazard is extremely uncertain

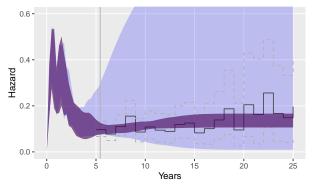
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# Survival modelling with external data: example



Registry data give annual survival rates from 5 to 25 years.

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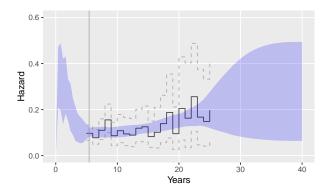


Registry data give annual survival rates from 5 to 25 years.

 Enables a confident "extrapolation" of the hazard for control group survival

Christopher Jackson

# Survival modelling with external data: example



Extrapolating up to 40 years, need longer-term data...

Can supply a known background hazard  $h_b(t)$ 

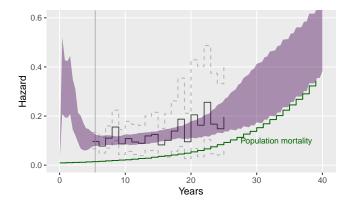
 e.g. national mortality statistics for people of comparable age/sex

Hazard for trial patients modelled as additive:

$$h(t) = h_b(t) + h_e(t)$$

i.e. no lower than population hazard  $h_b()$  assumed known, M-spline model placed on excess hazard  $h_e()$ 

### Additive hazards model



Suppose we judge by 40 years, survival will be similar to general population (though no evidence of this here!)

▶ i.e. (conditional annual) survival probability 0.72

We might elicit a distribution (Beta, say) with

mean 0.72

▶ 95% credible interval of 0.69–0.75 (say)

Equivalent information to having observed 724 survivors out of 1000 people

▶ posterior we would get with these data and an uniform prior. Bigger denominator → narrower credible interval

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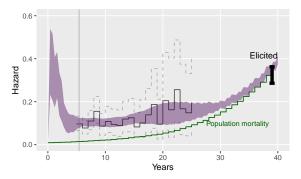
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# Model including long term expert judgement



Full evidence synthesis: trial, registry, population and elicited data all combined

**Christopher Jackson** 

Common to extrapolate survival by assuming disease-specific hazard diminishes to zero.

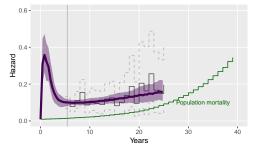
Mixture cure model:  $S(t) = p + (1 - p)S_0(t)$ 

Long-term survival converges to p (probability of "cure"), excess hazard converges to 0,  $S_0$  is survival for "uncured" person.

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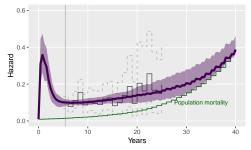


Little evidence of "cure" in data.

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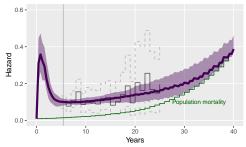


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#### More transparent to express "cure" assumption using elicited data

Christopher Jackson

Proportional hazards model

Flexible non-proportional hazards model (hazard shape parameters  $p_i$  depend on covariates)

Additionally can impose a waning effect when predicting

- log hazard ratio for a treatment decreases from modelled value to zero over some time period
- used for sensitivity analysis to what happens after a trial

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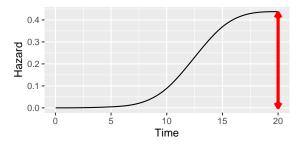
# Use simulation to determine consequences of a prior on an interpretable scale

- Simulate hazard curves from prior predictive distribution
- Find e.g. 10% and 90% quantile of h(t) over time t
- Leads to a distribution for the ratio between high and low values
- Find prior that, gives belief such as characterized and a final prior that, gives belief such as characterized and the second se

# Prior distributions: general strategy

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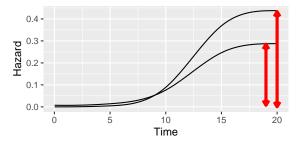
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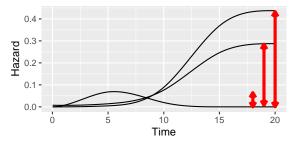
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# Final discussion

Further info in the paper, full doc and worked examples on https://chjackson.github.io/survextrap

#### Many details I haven't mentioned today, e.g:

- "Post-estimation" outputs e.g. restricted mean survival
- Model comparison by efficient cross-validation

Software status: "beta": not expected to change drastically before "full release" (CRAN), but more testing planned

#### Ongoing further work

- Simulation studies to assess if defaults are sensible
- Listen to users. Usable? Understandable? What is most challenging?

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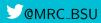
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## Thank you

MRC Biostatistics Unit



mrc-bsu.cam.ac.uk